

NAG Toolbox for MATLAB

s15ae

1 Purpose

s15ae returns the value of the error function $\text{erf}(x)$, via the function name.

2 Syntax

```
[result, ifail] = s15ae(x)
```

3 Description

s15ae calculates an approximate value for the error function

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt = 1 - \text{erfc}(x).$$

Let $\hat{x}$ be the root of the equation $\text{erfc}(x) - \text{erf}(x) = 0$ (then $\hat{x} \approx 0.46875$). For $|x| \leq \hat{x}$ the value of $\text{erf}(x)$ is based on the following rational Chebyshev expansion for $\text{erf}(x)$:

$$\text{erf}(x) \approx x R_{\ell,m}(x^2),$$

where $R_{\ell,m}$ denotes a rational function of degree ℓ in the numerator and m in the denominator.

For $|x| > \hat{x}$ the value of $\text{erf}(x)$ is based on a rational Chebyshev expansion for $\text{erfc}(x)$: for $\hat{x} < |x| \leq 4$ the value is based on the expansion

$$\text{erfc}(x) \approx e^{x^2} R_{\ell,m}(x);$$

and for $|x| > 4$ it is based on the expansion

$$\text{erfc}(x) \approx \frac{e^{x^2}}{x} \left(\frac{1}{\sqrt{\pi}} + \frac{1}{x^2} R_{\ell,m}(1/x^2) \right).$$

For each expansion, the specific values of ℓ and m are selected to be minimal such that the maximum relative error in the expansion is of the order 10^{-d} , where d is the maximum number of decimal digits that can be accurately represented for the particular implementation (see x02be).

For $|x| \geq x_{\text{hi}}$ there is a danger of setting underflow in $\text{erfc}(x)$. For $x \geq x_{\text{hi}}$, s15ae returns $\text{erf}(x) = 1$; for $x \leq -x_{\text{hi}}$ it returns $\text{erf}(x) = -1$.

4 References

Abramowitz M and Stegun I A 1972 *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

Cody W J 1969 Rational Chebyshev Approximations for the Error Function *Math.Comp.* **23** 631–637

5 Parameters

5.1 Compulsory Input Parameters

1: **x – double scalar**

The argument x of the function.

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

1: **result** – double scalar

The result of the function.

2: **ifail** – int32 scalar

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

There are no failure exits from s15ae. The parameter **ifail** has been included for consistency with other functions in this chapter.

7 Accuracy

See Section 7 of the document for s15ad.

8 Further Comments

None.

9 Example

```
x = -6;  
[result, ifail] = s15ae(x)  
  
result =  
    -1  
ifail =  
      0
```